WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS METHODS
SEMESTER ONE 2018
TEST 1 EXPONENTIALS AND LOGARITHMS AND DIFFERENTIAL CALCULUS

Tuesday 27th February

Name: _____

Time: 45 minutes **Part A:** ___/29 **Part B:** ___/14 **Total:** ___/43 **%**

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet for both sections, and an A4 page of notes, plus up to 3 permitted calculators in the Calculator Allowed section.

Topic	Confidence
Exponentials and logarithms	
• Logarithm laws and solving equations	Low Moderate High
• Logarithmic graphs and scales	Low Moderate High
• Exponential growth and decay	Low Moderate High
Differential calculus	
• Exponential functions	Low Moderate High
• Natural logarithmic functions	Low Moderate High
• Differentiation rules	Low Moderate High

Self reflection (eg. comparison to target, content gaps, study and work habits etc)

1. [10 marks]

Solve the following equations, giving exact answers in simplest form.

a) $2^{x-1} = 7$

[2]

b) $\log_4 x = \frac{3}{2}$

[2]

c) $\log_3(3x+1) = 2 + \log_3(x)$

[3]

d) $e^{2x} - e^x = 6$

[3]

2. [5 marks]

Given that $\log_2 3 = m$ and $\log_2 5 = p$,

a) express $\log_2 30$ in terms of m and p

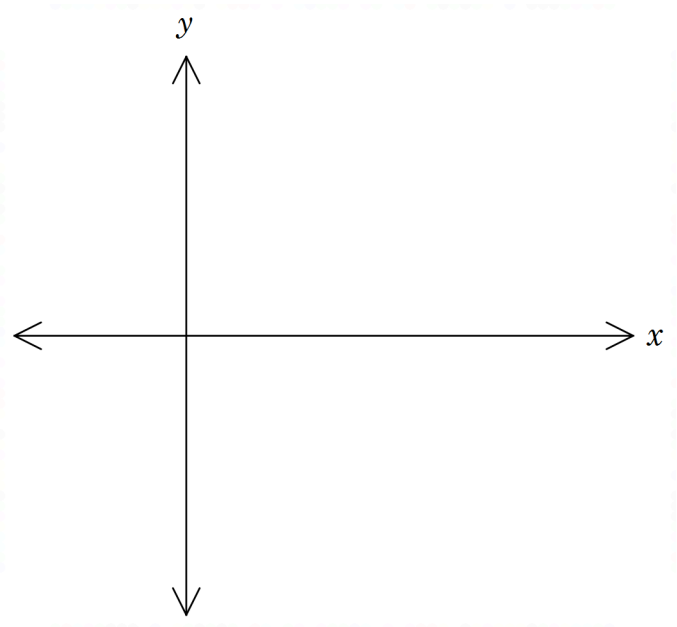
[2]

b) evaluate 2^{p-2m}

[3]

3. [5 marks]

a) Sketch the graph of $y = \log_2(x+2) - 1$ labelling asymptotes, intercepts and a key point.



[3]

b) Explain why graphs of equations of the form $y = \log_a(x+a) - 1 \quad a > 0$ always pass through the origin.

[2]

4. [9 marks]

a) Differentiate the following equations. Answers should be in the same form as the question but do not need to be simplified or expressed in factored form.

i) $y = \dfrac{x^3}{e^x}$

[2]

ii) $y = \sqrt{x^2 - 1}\left(5x - x^2\right)$

[3]

b) Given $y = \dfrac{u^3}{3} - u$ and $u = \ln(2x - 3)$ determine $\dfrac{dy}{dx}$ in terms of x.

[4]

Name: _____

Calculator Allowed Section **15 minutes** **/14**

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5. [9 marks]

The size of a population, W, is measured every year and has an instantaneous rate of change given by the equation $\frac{dW}{dt} = \frac{W}{20}$, where t is the number of years after recording commenced.

The initial population is 2500.

a) State whether the population is increasing or decreasing, giving a mathematical reason for your answer.

[2]

b) State an equation for W in terms of t.

[2]

c) Find the size of W when t = 4. (Round to the nearest integer).

[1]

d) Find the rate of change of W when t = 4, (correct to 2 decimal places).

[2]

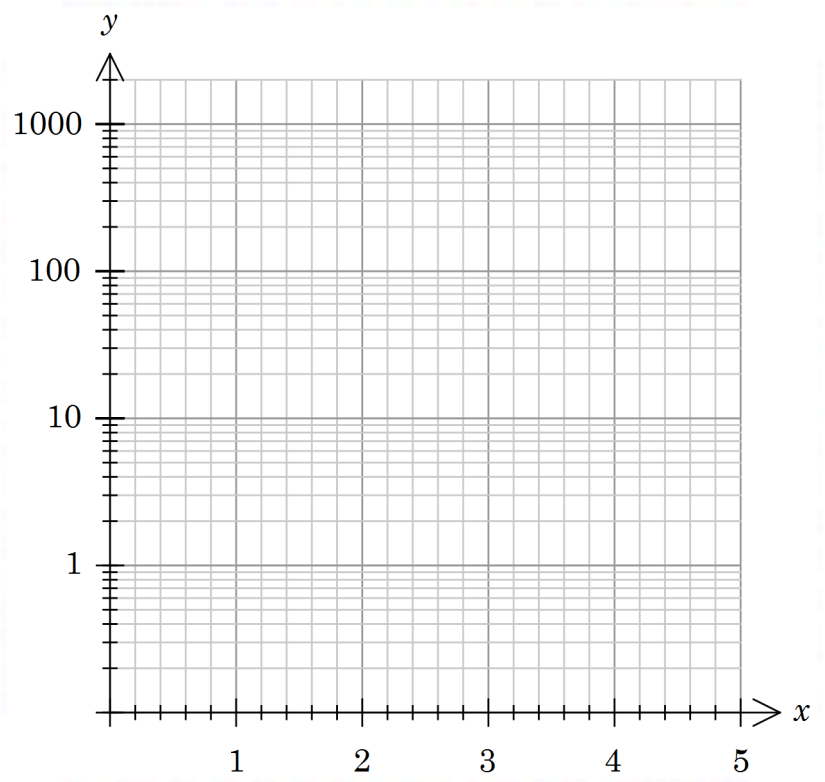
e) Determine the year in which the instantaneous rate of change first reaches 500 units per annum.

[2]

6. [5 marks]

a) Plot the function $y = 4^x$ on the axes below.

Note the logarithmic scale on the y-axis.



[2]

b) The 'db' or Decibel scale for sound level measuring loudness of sound is given by:

$$\text{Sound level} = 10\log\left(I \times 10^{12}\right) \text{ db}$$

Where I is the intensity of the sound in Watts per m^2

Show that doubling the intensity of a sound increases the sound level by only a few db.

[3]